Reduced Basis Finite-element Method for Electromagnetic Field Computation of Geometric Deformation Problems

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In view of parameterized nonlinear problems for electromagnetic field computation, when the geometric parameters change, a series of field computation need to be repeated, in which a lot of double computation lead to large amount of calculation. This paper introduces are reduced basis finite-element method (RBFEM) which is based on reduced basis matrix to solve the finite-element matrix equation. Since the RBFEM significantly reduces the dimension of the matrix equation, it accomplishes the entire process with less amount of calculation compared to traditional FEM. The RBFEM is employed to solve both small scale and large geometric deformations.

***Index Terms*—Finite-element method, geometric deformation, parameterized nonlinear problem, reduced basis method.**

I. Introduction

In computational electromagnetics, one fundamental problem is to develop methods for the solution on domains which contain several objects, and these objects may have complex and time-dependent geometry[1].

Finite-element method(FEM) is a numerical method based on variational principle and polynomial interpolation in the Sobolev space[2]. When obtaining discrete solutions to continuous problems described by partial differential equations (PDE) and boundary and initial conditions with FEM, it is necessary to generate a computational mesh to define the unknowns [2].

In order to improve the performance of electromagnetic devices, geometrical size optimization, shape optimization and topology optimization are all necessary in design processes. Mostly, successful optimal design is dependent on the accuracy of the mathematical model. Optimal design using finite-element analysis (FEA) has gradually become a regular design method for improving the performance of electromagnetic (EM) devices, subject to, for example, volume, weight and cost constraints. In electromagnetic field computation, researchers sometimes have to deal with the issue with large geometric deformation, for example, movement with long distance. The calculation is complicate and it often causes abundant of working load.

One of the approaches to reduce the computing load is to improve the algorithm of FEM. In this paper, a reduced basis method (RBM) is presented. The RBM is applied for both small scale deformation and large geometric deformation. Comparisons between traditional FEM and the reduced basis finite element method (RBFEM) for the deformation problems are conducted. The results show that the RBFEM is feasible and is able to save computing time.

II. Reduced Basis Finite-Element Method

The reduced basis method (RBM) [3]is one of approximate analysis methods. Its basic principle is to solve the FEM equations after obtaining the initial solution of u, when the modification of design parameters occurs, instead of getting the solution u* by directly calculation of the FEM equations, RBM finds u* through the relationship between u* and u. Thereby it can significantly reduce the amount of calculation.

The differential equation of 2-D magnetic field is

$$\frac{\partial}{\partial x}(\upsilon\frac{\partial A}{\partial x})+\frac{\partial}{\partial y}(\upsilon\frac{\partial A}{\partial y})=-J \tag{1}$$

In equation (1) A is the component of magnetic vector potential along the z axis. J is the electric current density along the z axis.

After applying the FEM to nonlinear magnetic problem, the matrix equation (1) on the entire solution domain is

$$Au=b \tag{2}$$

In (2), A is the initial coefficient matrix, u is the initial magnetic potential, supposing that the number of total degrees of freedom (DOF) is n. In nonlinear problems, when the parameters $p_1, p_{2,\dots,} p_m$ have been varied, the equation (2) needs to be calculated repeatedly, and the computing load is heavy.

Based on reduced basis (RB) technology, the magnetic vector potential u* can be expressed by using the linear combination of linear independent vector u_1, u_2, ..., u_m, where m is the number of parameters, it is far less than n.

$$u^*=Zv \tag{3}$$

where $Z=\{z_1, z_{2,\dots,} z_m\}$ is the RB matrix, and $v=[v_1, v_2, \dots, v_m]^{\mathrm{T}}$ is an unknown coefficient vector. Substituting (3) into (2), we have

$$A^*Zv=b \Rightarrow Z^{\mathrm{T}}A^*Zv=Z^{\mathrm{T}}b \tag{4}$$

Equation (4) can be written as

$$A^*v=b^* \tag{5}$$

$A^*=Z^{\mathrm{T}}A^*Z$ is the $n\times m$ order RB coefficient matrix. b* is the n×1 order RB right hand item. When v is solved, using the equation (3) one can get the next iteration magnetic potential vector u*.

Construction of the RB matrix is an essential step which can affect the working load. Traditional RBF requires that the z_i are linearly independent vectors. Basic technique is to use the u_i as the z_i. However this technique is complicate. In this paper, a new strategy for constructing the RB matrix is proposed.

This paper employs the Taylor series expansion of the magnetic vector potential, which is obtained after the modification of geometric parameters, in the vicinity of the initial magnetic vector potential. Due to the solution of second-order derivative and above is complicated and time-consuming, here we only take the first-order derivative into consideration.

$$u^* = u + \sum_{i=1}^{m} (p_i^* - p_i^0) \frac{\partial u}{\partial p_i} \tag{6}$$

To the both sides of equation (2), we do the first order partial derivatives to p_i.

$$A \frac{\partial u}{\partial p_i} = \frac{\partial b}{\partial p_i} - \frac{\partial A}{\partial p_i} u \tag{7}$$

As b is a constant, we have

$$\frac{\partial b}{\partial p_i} = 0 \quad , \quad A \frac{\partial u}{\partial p_i} = -\frac{\partial A}{\partial p_i} u \tag{8}$$

In FEM the region is divided into many units. The total coefficient matrix is the superposition of the coefficient matrix of each unit. Assuming that the number of elements is n_e, the equation (6) can also be expressed as

$$A \frac{\partial u}{\partial p_i} = -\sum_{j=1}^{n_e} \frac{\partial A_j^e}{\partial p_i} u_j^e \tag{9}$$

Considering the Taylor series expansion, the RB can be chosen as:

$$z_1 = u, \quad z_i = \frac{\partial u}{\partial p_i}, (i = 1, \ldots, N) \tag{10}$$

The calculation process of solving the equation (4) is replaced by solving the equation (5)

III. Numerical Examples

The RBM is applied to two types of nonlinear problems which involves small scale deformation and large geometric deformation.

A. Small Scale Deformation Problem

Considering that the variation only occurs in a limited area, limited number of elements in A is changed in the iteration. In RBFEM, the coefficient matrix A can be divided into two parts, stabilized A_0 and parameterized A_p.

$$A = A_0 + A_p \tag{11}$$

The application example is to optimize the geometrical sizes of an electromagnetic brake. The geometry and design parameters are shown in Fig. 1. There are three geometry variables, 10mm<p_1<25mm, 23mm<p_2<28mm, and 21mm<p_3<26mm.

The flux lines of the initial model and the model when parameters are varied are shown in Fig. 2. The Comparison of computing time of one iteration between traditional FEM and RDFEM with different number of unknown magnetic potentials is shown in Table I.

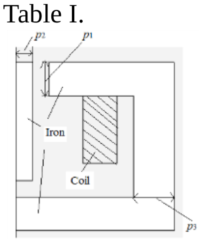

Fig. 1.The geometry and its parameters.

(a) (b)

Fig. 2. Flux lines before and after variation. (a)Initial flux lines. (b) Flux lines after variation

TABLE I
Comparison of Computing Time of Solving FEM Matrix Equation

	Computing time (s)		
Method	n=6327	n=13027	n=89871
FEM	23	88	136
RBFEM	25	76	102

B. Large Geometric Deformation Problem

In order to verify that the RBFEM is practicable for electromagnetic problems with large geometric deformation, a simple electromagnetic field problem as shown in Fig. 3 is tested. The parameters are 0mm<p_1<100mm, 0mm<p_2<100mm, and 3mm<p_3<10mm. The flux lines of the initial model and the model when parameters are varied are shown in Fig. 4.

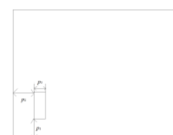

Fig. 3.The geometry and its parameters.

(a) . (b)

Fig. 4.Flux lines before and after variation. (a)Initial position. (b) End position.

TABLE II
Comparison of Computing time of Solving FEM Matrix Equation

	Computing time (s)		
Method	n=5413	n=14138	n=86342
FEM	31	93	172
RBFEM	29	81	123

IV. Conclusion

The aim of this study is to apply the RBFEM to nonlinear electromagnetic problems which involve geometric deformation. The results of the calculation and the comparison between FEM and RBFEM show that RBFEM is able to accelerate the computing process.

References

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